A Beginner's Guide to Quantum Error Correction

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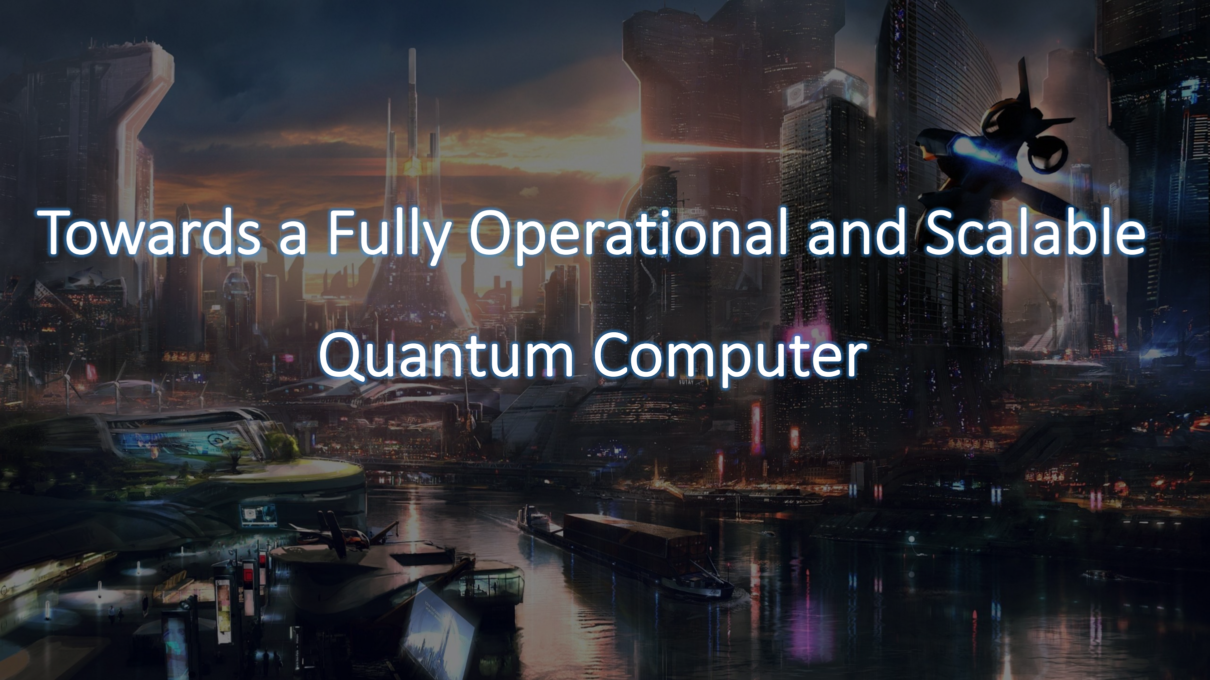
Towards a Fully Operational and Scalable Quantum Computer



Quantum Decoherence

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- The loss of quantum coherence.
  - ➢ the loss of information from a system into the environment.
- Present in the transmission, processing, or storage of quantum information.

Quantum Decoherence

Towards a Fully Operational and Scalable Quantum Computer

- **Understand environmental decoherence processes and model them properly.**

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- **Understand environmental decoherence processes and model them properly.**
- **Use error correction to protect quantum information against decoherence.**

Model a Noisy Quantum System

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$$\epsilon : \rho \longrightarrow \sum A_k \rho A_k^\dagger$$

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Binary Symmetric Channel (BCS_p)

IN: $b \in \{0, 1\}$

OUT: $\begin{cases} b, & \text{prob. } 1 - p \\ \neg b, & \text{prob. } p \end{cases}$

Assume $p \in [0, 1]$, the channel behaves independently for each bit that passes through it.

Examples of Single-Qubit Errors

Bit Flip X : $X|0\rangle = |1\rangle, X|1\rangle = |0\rangle$.

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Phase Flip Z : $Z|0\rangle = |0\rangle, Z|1\rangle = -|1\rangle$.

$$Y = iXZ = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}.$$

Complete Dephasing : $\rho \longrightarrow 1/2(\rho + Z\rho Z^\dagger)$.

Rotation R_θ : $R_\theta|0\rangle = |0\rangle, R_\theta|1\rangle = e^{i\theta}|1\rangle$.

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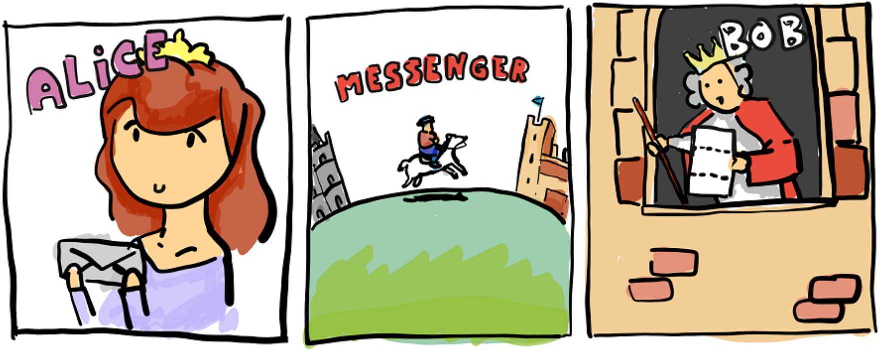
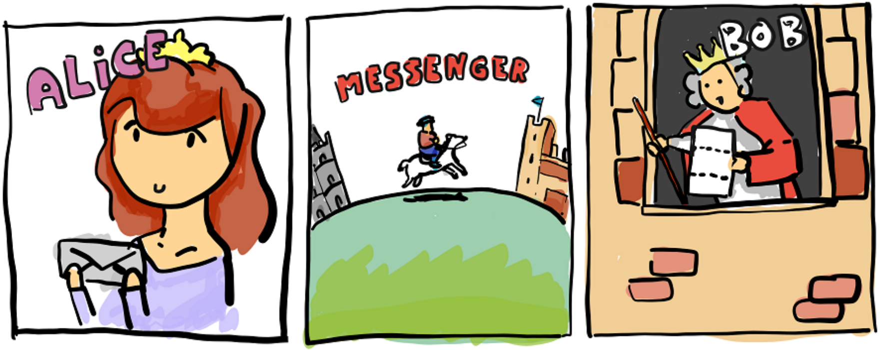
Definition

A single-qubit Pauli error could be one of the following single-qubit errors:

- A bit-flip error X;
- A phase-flip error Z;
- Both a bit-flip and a phase-flip error: Y.



Suppose Alice wants to communicate to Bob, but their communication channel is noisy. How can they **reduce the noise level**?



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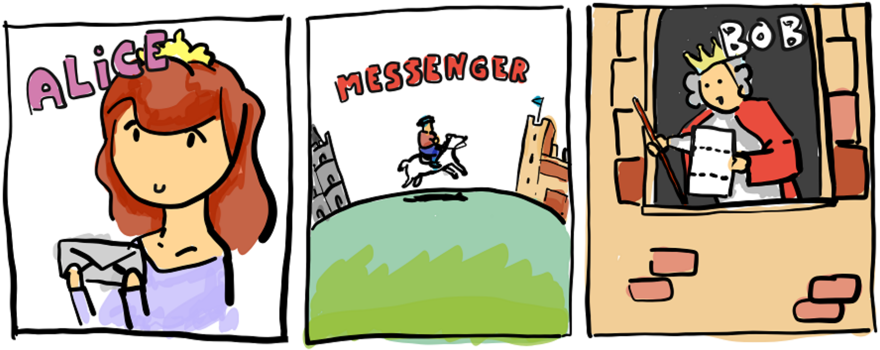
1. Get a better communication channel (*BCS* with a smaller p).



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1. Get a better communication channel (BCS with a smaller p).
2. **Use quantum error correction codes.**

3-Bit Repetition Code

1. Alice encodes $b \in \{0, 1\}$ as bbb, and sends the three bits through the channel.

$$0 \longrightarrow 000 \qquad 1 \longrightarrow 111.$$

2. Bob decodes the three bits he receives by taking the majority count.
   The three bits that Bob receives may not be the same.

$$000 \longrightarrow \begin{cases} 100, \text{The 1st bit is flipped;} \\ 010, \text{The 2nd bit is flipped;} \\ 001, \text{The 3rd bit is flipped.} \end{cases} \qquad 111 \longrightarrow \begin{cases} 011, \text{The 1st bit is flipped;} \\ 101, \text{The 2nd bit is flipped;} \\ 110, \text{The 3rd bit is flipped.} \end{cases}$$

3. If no more than one bit is flipped, this method succeeds because flipping one bit does not change the majority.

Obstacles in Quantum Error Correction

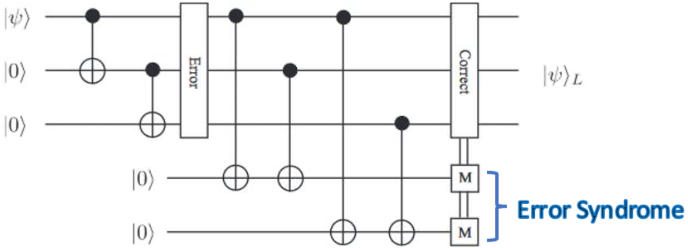
- No-cloning theorem forbids the classical repetition strategy.
- Measuring qubits to identify errors would collapse superpositions.
- Need to correct bit flip and phase errors.
- Need to handle continuous rotations, decohering maps, etc.
- Need to correct errors on multiple qubits.

[1] Daniel Gottesman: Quantum Error Correction and Fault Tolerance (Part 1) - CSSQI 2012.

Correct a Bit Flip Error

To correct a single bit flip error, we can encode the data as:

$$0 \longrightarrow 000, 1 \longrightarrow 111$$



If there is a single bit flip error, we can correct the state by choosing the majority of the three bits.

Analysis

State that Bob receives	**Probability**	**Syndrome**	**Correction**
$(\alpha \lvert 100\rangle + \beta \lvert 011\rangle) \lvert 11\rangle$	$p(1-p)^2$	11	Flip the 1st qubit
$(\alpha \lvert 011\rangle + \beta \lvert 100\rangle) \lvert 11\rangle$	$p^2(1-p)$	11	Flip the 2nd and 3rd qubits

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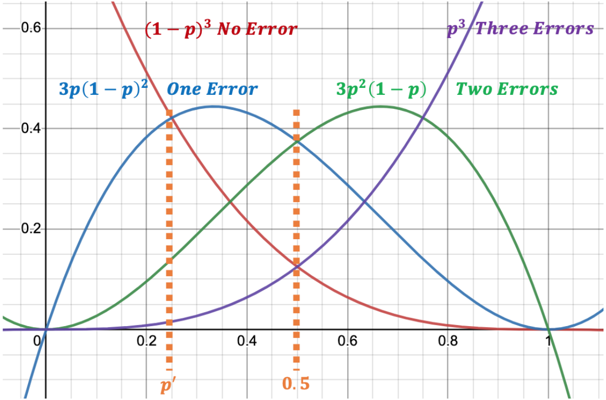
Suppose Bob measures 11, then he must either receives

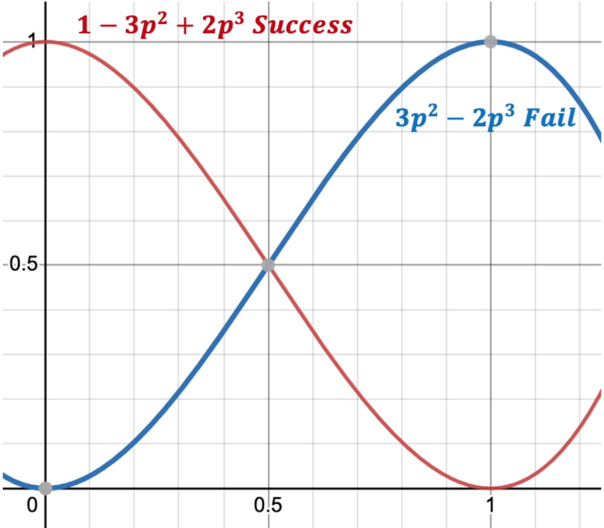
- $\alpha \lvert 100\rangle + \beta \lvert 011\rangle$ with probability $p(1-p)^2$, or
- $\alpha \lvert 011\rangle + \beta \lvert 100\rangle$ with probability $p^2(1-p)$.

When errors are rare, one error is more likely than two errors.



Repetition Code Corrects up to One Error

Error	Probability	Success/Fail
0	$(1-p)^3$	$1-3p^2+2p^3$
1	$3p(1-p)^2$	
2	$3p^2(1-p)$	$3p^2-2p^3$
3	$(1-p)^3$	



Summary

Encoding: $\alpha|0\rangle + \beta|1\rangle \longrightarrow \alpha|000\rangle + \beta|111\rangle \neq (\alpha|0\rangle + \beta|1\rangle)^{\otimes 3}$

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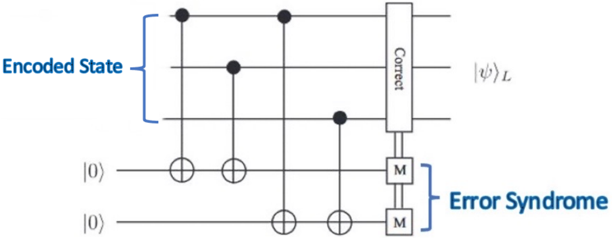
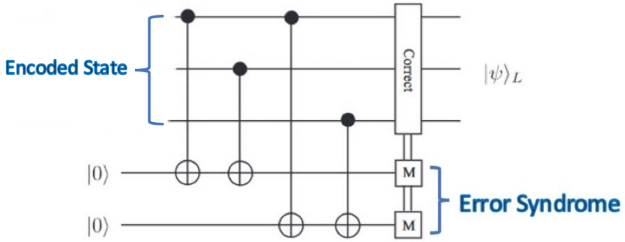
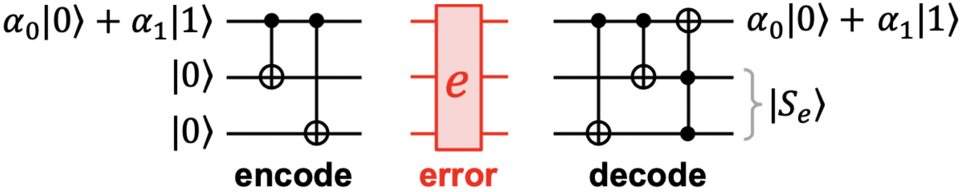
Detect the error: Measure the error, not the data.







[1]Cleve, R. (2021). Introduction to Quantum Information Processing. Retrieved June 6, 2023.

Correct a Phase Flip Error

Can we use the same procedure to correct a single-qubit phase flip error?

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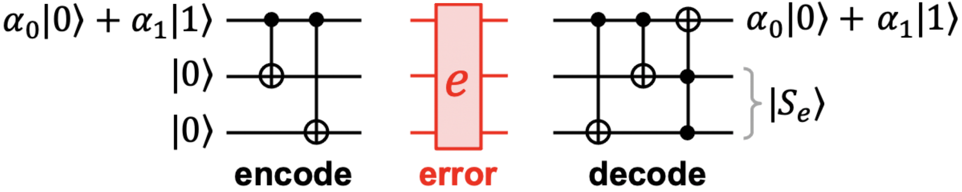

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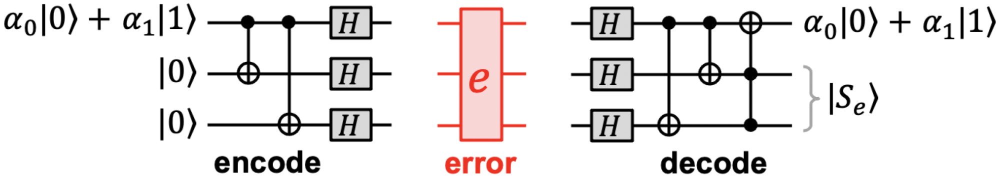
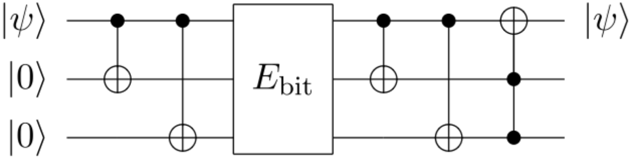
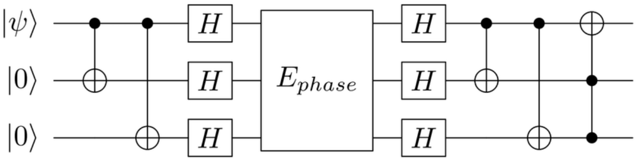


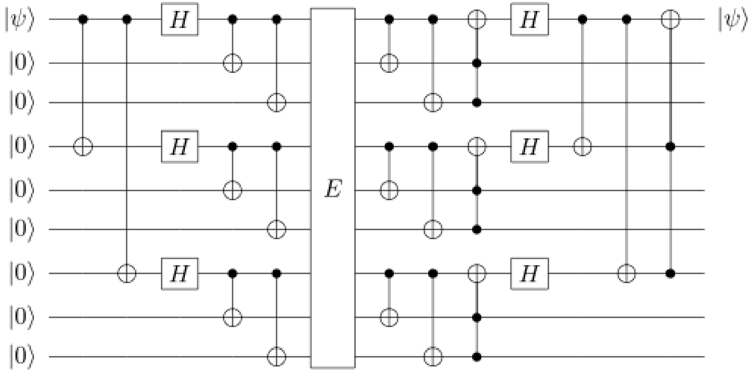
Correct a Phase Flip Error

Since $HZH = X$, we can reduce the problem of the phase flip error correction to an instance of the bit flip error correction.





Correct a Single-Qubit Pauli Error



Shor's Nine-Qubit Code

Welcome to the error correction zoo

Jump to ▸ Linear binary, Additive q-ary, RS, RM, LDPC, Polar, Rank-metric, STC, Stabilizer, CSS, Good QLDPC, Kitaev surface, Color, Topological, Holographic, EAQECC, GKP, Cat

Classical Domain ▸ Binary Kingdom, Galois-field Kingdom, Matrix Kingdom, Lattice Kingdom, Spherical Kingdom, Ring Kingdom, Group Kingdom
Quantum Domain ▸ Qubit Kingdom, Modular-qudit Kingdom, Galois-qudit Kingdom, Bosonic Kingdom, Fermionic Kingdom, Spin Kingdom, Group Kingdom, Category Kingdom

Code lists ▸ Approximate quantum codes, Binary linear codes, Quantum CSS codes, Codes with notable decoders, Dynamically generated quantum codes, Asymptotically good QLDPC codes, Hamiltonian-based codes, Holographic codes, Quantum codes based on homological products, LDPC codes, MDS codes, Perfect codes, q-ary linear codes, Quantum LDPC codes, Quantum codes with code capacity thresholds, Quantum codes with fault-tolerant gadgets … (see all)

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Code lists
All codes
Glossary of concepts

css

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Binary linear code defined on edges on a regular graph G such that each subsequence of bits corresponding to edges in the neighborhood any vertex belong to some \textit{short} binary linear code C_0. Expansion properties of the underlying graph can yield efficient decoding algorithms. More …

Stats at a glance: **275** code entries, **15** kingdoms, **2** domains, **72** classical codes, **124** quantum codes, **79** abstract property codes, **27** topological codes, **33** CSS codes, **44** quantum LDPC codes, and **26** bosonic codes.

Thanks!