



# **LOGIC & PUZZLES**

**NOVA SCOTIA MATH CIRCLES JANUARY 2021**  
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# LOGIC

- **WHAT IS LOGIC?**
- **WHAT IS A PROPOSITION?**
- **WHAT IS A TRUTH TABLE?**
- **WHAT ARE LOGIC CONNECTIVES?**

# WHAT IS LOGIC?

**Logic** is a systematic way of thinking that allows us to parse the meanings of sentences and to deduce new information from old information.

## Example 1

1. If the weather is nice this Thursday, we will go hiking.
2. The weather is nice this Thursday.
3. We will go hiking.

## Example 2

1. The side of square  $X$  is of 4 cm.
2. If any square has side length  $a$ , then its area is  $a^2$ .
3. Square  $X$  has area  $16 \text{ cm}^2$

# WHAT IS A PROPOSITION?

- A **proposition** is a sentence or a mathematical expression that is either definitely true or definitely false.
- To prove that a proposition is true, we use logic to help us understand statements and to combine pieces of information to produce new pieces of information.



# ATOMIC PROPOSITIONS

- An **atomic proposition** is a proposition that cannot be broken down into smaller concepts.
- **Example**
  - My car is grey.
  - I live in Halifax.
  - Dogs like ice-cream.



# COMPOUND PROPOSITIONS

- A **compound proposition** is a proposition that is built of more than one atomic proposition.
- **Examples**
  - My car is grey and it is electric.
  - I live in Halifax or I live in Shanghai.
  - Pigs can fly but dogs can swim.



# LOGIC CONNECTIVES

- The “joints” we use to construct compound statements.



And



Or



NEGATION



IMPLICATION

- **Truth table** tells us how to compute the truth value of a compound statement.
- The **truth value** of a proposition is true if the proposition is true and false otherwise.

# WHAT IS A TRUTH TABLE?

## AND

$P$	$Q$	$P \wedge Q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$F$
$F$	$F$	$F$

## OR

$P$	$Q$	$P \vee Q$
$T$	$T$	$T$
$T$	$F$	$T$
$F$	$T$	$T$
$F$	$F$	$F$

## Not

$P$	$\sim P$
$T$	$F$
$F$	$T$



# WHAT IS AN IMPLICATION?

- There is another way to combine two statements. Given two statements  $P$  and  $Q$ , we can form the new statement “If  $P$  then  $Q$ ” or “ $P$  implies  $Q$ ”, written as “ $P \Rightarrow Q$ ”.
- What does the truth table of “ $P \Rightarrow Q$ ” look like? Can you explain it?

$P$	$Q$	$P \Rightarrow Q$
$T$	$T$	$T$
$T$	$F$	$F$
$F$	$T$	$T$
$F$	$F$	$T$



# TRUTH TELLERS & LIARS (TTTL)

- **TTL A: CHOPPY LIARS**
- **TTL B: CUNNING LIARS**
- **TTL C: KNIGHTS, KNAVES, AND SPIES**



# TRUTH TELLERS & LIARS A

**TTL A: CHOPPY LIARS**

# TTL A: ACTION 1

*This sentence and the next one are true.*

*This sentence has exactly three words in it.*

Is the first sentence above true or false?



I am older than 15,  
and not older than 15.



## **TTL A: ACTION 2**

Dilshad makes the statement on the right. Is her statement truthful or lying?

# TTL A: ACTION 3

Mia makes the statement on the right. If she is lying, then which of the situations below are possible?

**Situation 1:** Mia went to the store and the theater.

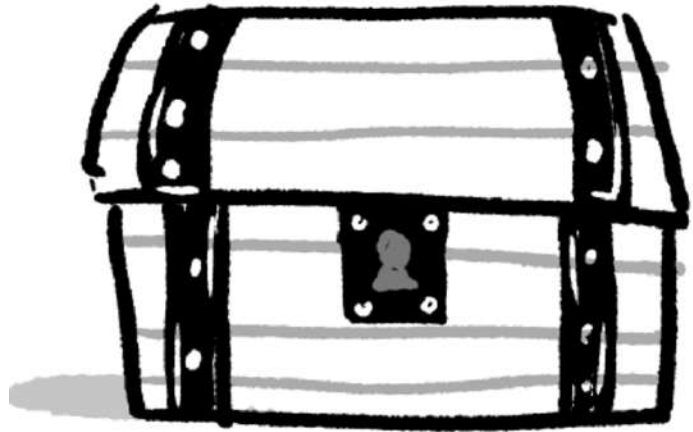
**Situation 2:** Mia went to the store but not the theater.

**Situation 3:** Mia went to the theater but not the store.

**Situation 4:** Mia didn't go to the theater and he didn't go to the store.

I went to the store  
and the theater.





# **TTL A: ACTION 4**

**IF A BOX IS NOT NOT NOT NOT OPEN, IS IT  
OPEN OR CLOSED?**

# TTLA: ACTION 5

Suppose you are visiting an island with knights, who always tell the truth, and knaves, who always lie.

Which statement is impossible for an islander to make?

- I am either a knight or a knave.
- I am a knight.
- I am a knave.





# **TRUTH TELLERS & LIARS B**

**TTL B: CUNNING LIARS**

# TTL B: ACTION I

- Suppose you are visiting an island with sknights and sknaves.
- Sknights tell the truth on every day but Saturday, on which they always lie. Sknaves lie on every day but Saturday, on which they always tell the truth.
- You don't know what day of the week it is. You meet an islander and ask, "Is it Saturday?" They respond "Yes."

What type must the islander be?

- A sknight
- A sknave
- It is impossible to determine

# TTL B: ACTION 2

Alice, Bob, Charlie, and David have all been invited to a party.

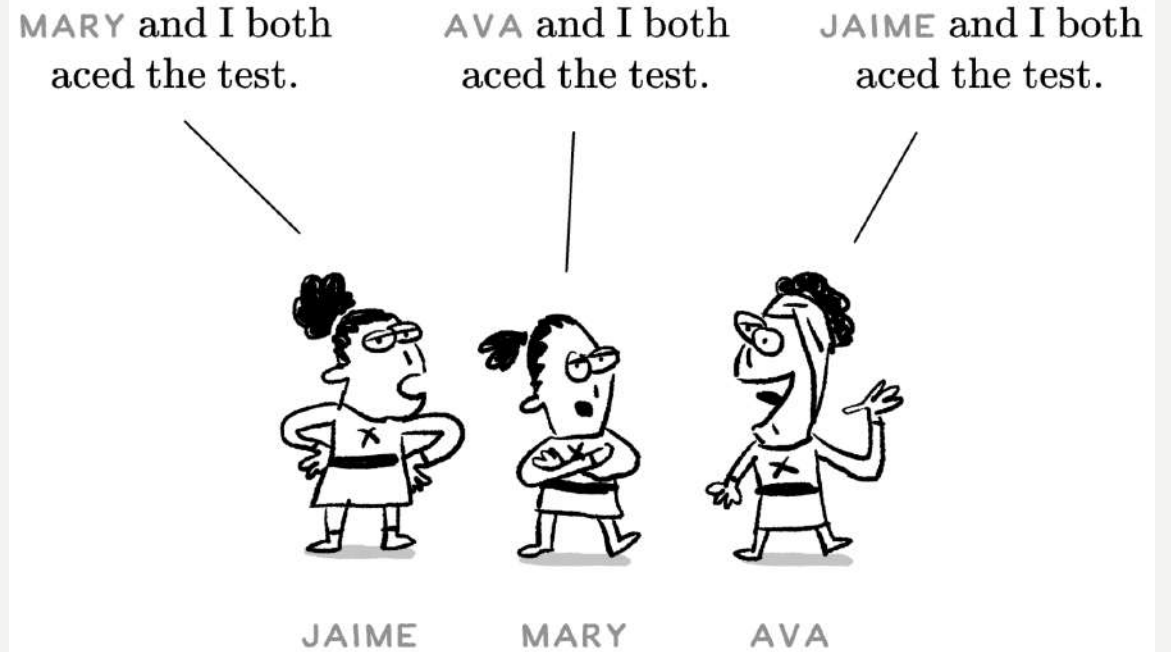
Charlie says that he will go to the party if (and only if) Alice **and** Bob are going to the party. David says that he will go to the party if (and only if) Alice **or** Bob is going to the party. **If Alice decides to go to the party but Bob decides not to go to the party, which is true?**

- A. Charlie and David both go to the party.
- B. Charlie goes to the party, but David does not.
- C. Charlie does not go to the party, but David does.
- D. Charlie and David both do not go to the party.



# TTL B: ACTION 3

- Three kids named Jamie, Mary, and Ava talked to their parents after taking a test. Each one either aced the test or didn't, and the kids are aware of how each other did.
- Of the four statements in the multiple choice options, which of the situations is not possible? That is, the other three are possible, which one is not?
- **HINT:** Go through the cases if 0, 1, 2, or all 3 children aced the test.



a. All of them are telling the truth

b. Exactly 2 of them are telling the truth

c. Exactly 1 of them is telling the truth

d. None of them are telling the truth

I am the younger or I  
am the truth teller.



I am the older and I  
am the truth teller.



## TTL B: ACTION 4

The two children on the left are not the same age. One of them is telling the truth and the other is lying. Who is the truth-teller?

Note: Remember, our use of “or” is “inclusive or”; a statement which uses “or” is true so long as at least one of the two parts is true.

# TTL B: ACTION 5

- Each person on the left makes all three statements below.
- **Statement 1:** "My hat is red and I have a mustache."  
**Statement 2:** "I have a mustache or a beard."  
**Statement 3:** "I have no beard and my hat is green."
- For which of the people are all three statements false?



A



B



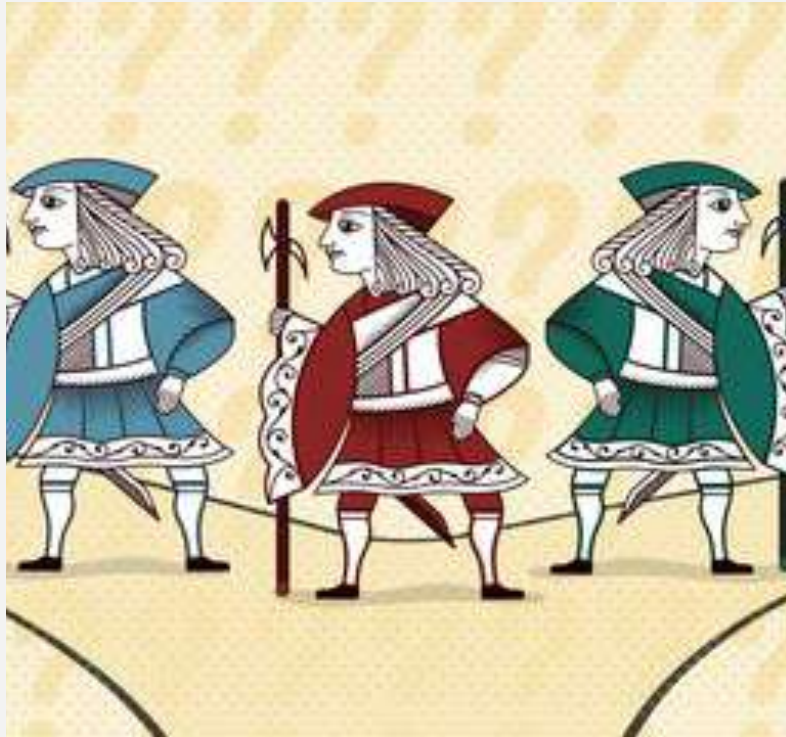
C



# TRUTH TELLERS & LIARS C

TTL C: KNIGHTS, KNAVES, AND SPIES

# KNIGHTS, KNAVES, AND SPIES



A puzzle is defined by the statements of three people. Out of those three people, one is a **knight** who always tells the truth, one is a **knave** who always lies, and one is a **spy** who can either lie or tell the truth.

Each statement presented here admits one valid assignment, and this assignment is unique.



# HOW TO PLAY?

- There are twenty statements in the handout. Each statement gives a puzzle where you need to find the unique valid assignment of the knight, knave, and spy among A, B, and C.
- Your goal is solve as many puzzles as possible within 15 mins.
- Write your solutions in the following format:
  - Let F=Knave, T=Knight, S=Spy.
  - For example, FTS means that person A could be a knave, person B could be a knight, and person C could be the spy. Similarly SFT means person A could be a spy, person B could be a knave, and person C could be a knight.
  - An empty line after the puzzle means there is no valid assignment.



# WEREWOLVES

- WEREWOLVES A
- WEREWOLVES B
- WEREWOLF C

**DON'T HESITATE TO ASK FOR HINTS!**

# WEREWOLVES A



- A certain village has 3 inhabitants, each either human or werewolf. Humans always tell the truth and werewolves always lie. They each make a statement.
- Advik says, "**At least one of us is a werewolf.**"
- Bardia says, "**At least one of us is a human.**"
- Cherry says, "**Exactly two of us are werewolves.**"

## WHAT MUST BE TRUE?

- Only Advik is a werewolf
- Only Bardia is a werewolf
- Only Cherry is a werewolf
- Two of them are werewolves
- All three are werewolves
- None of them are werewolves

# WEREWOLVES A



- A certain village has 4 inhabitants. Humans always tell the truth and werewolves always lie. They each make a statement.
- Denali says, "**Ezra is a human.**"  
Ezra says, "**Flynn is a human.**"  
Flynn says, "**Denali is a human.**"  
Garnet says, "**At most 2 of us are humans.**"
- Unfortunately, this isn't enough information to find any werewolves. Sadly, the consequence is that poor Garnet is found killed by werewolves outside the village the next day. Given that Garnet was clearly a human, how many out of the remaining villagers (out of Denali, Ezra, and Flynn) are werewolves?

## WHAT MUST BE TRUE?

- Only I is a werewolf
- Exactly two are werewolves
- All three of them are werewolves
- There isn't enough information to know for certain.

# WEREWOLVES B



## HOW MANY OF INHABITANTS ARE WEREWOLVES?

A certain village has 100 inhabitants. Humans always tell the truth and werewolves always lie.

Every single one of the inhabitants makes the statement, "At least one of us is a human."

If you also know that at least one of the inhabitants is a werewolf, how many of them are werewolves?

- Exactly one is a werewolf
- Exactly 50 are werewolves
- Exactly 99 are werewolves
- All of them are werewolves

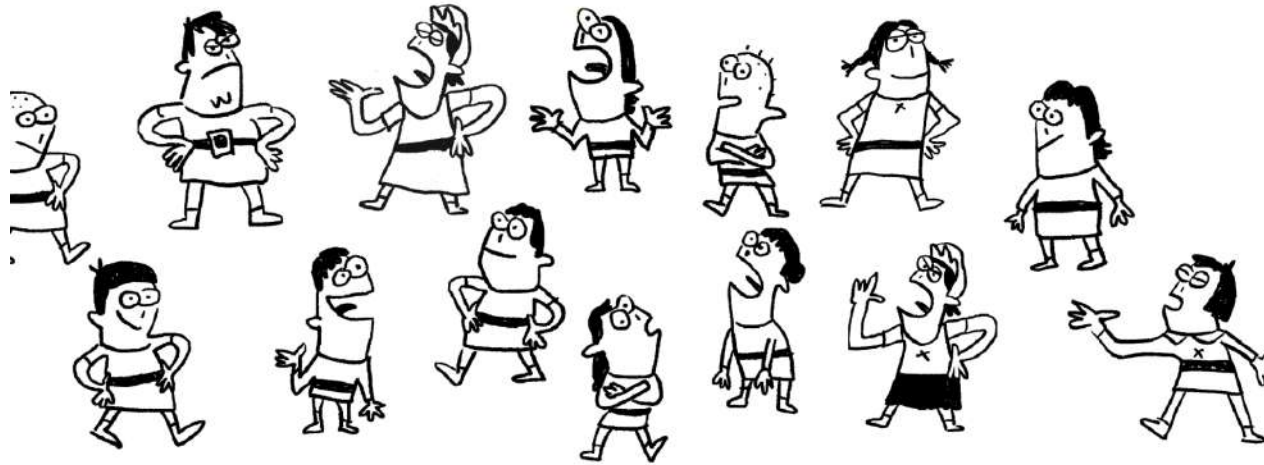
# WEREWOLVES B

- A certain village has 4 inhabitants. Humans always tell the truth and werewolves always lie, with exactly one exception: a **collaborator** who is a human working with the werewolves. The collaborator will always lie (but is otherwise considered human). Each inhabitant of the village makes two statements:
- Harry says, "**Exactly one of us is a human. Jota is the collaborator.**"
- Iris says, "**Exactly two of us are human. I'm not the collaborator.**"
- Jota says, "**Exactly three of us are human. Kori is the collaborator.**"
- Kori says, "**Exactly four of us are human. Iris is the collaborator.**"

**WHO IS THE COLLABORATOR?**



# WEREWOLF C



## IS PERSON #1 A HUMAN OR A WEREWOLF?

- Person #1 is a human
- Person #1 is a werewolf
- Lack of information to decide

A certain village has 12 inhabitants. Humans always tell the truth and werewolves always lie. They each make a statement.

Person #1: "Person #2 is a werewolf."

Person #2: "Person #3 is a werewolf."

Person #3: "Person #4 is a werewolf."

This pattern continues until Person #12.

Person #12: "There are at most 6 werewolves."



**LOGIC  
REASONING  
CHALLENGE**



# PROPOSITIONAL LOGIC

- Consider the following three statements:
  - I. Those who like paintings like flowers.
  - II. Those who like running like music.
  - III. Those who do not like music do not like flowers.
- **If these three statements are all true, which of the following statements must be true?**
  - A. Those who like running like flowers.
  - B. Those who like paintings like music.
  - C. Those who like flowers do not like running.
  - D. Those who like running do not like paintings.
  - E. Those who like paintings like running.



# HAT COLOR PUZZLE



- Three people are in a room wearing hats.
- Two of them are wearing blue hats and one of them is wearing a red hat; they can't see what color hat they are wearing, but they can see everyone else's hats.
- They are playing a game where their objective is to figure out the color of their own hat. (You can assume with this question and any other in this quiz the players will know how many hats there are and in which colors.)
- **If a player sees the other two players have a red hat and a blue hat, what color hat are they wearing?**
  - A. Blue
  - B. Impossible to tell
  - C. Red

# MIND READING

- Think of a number. Multiply it by 2 and add 4. Multiply the result by 0.5 and add 3, and then subtract the original number. I know what number you have left. What must it be?



# LOGIC REASONING

- John, Aries, and Joseph are brothers with different ages. **Who is the youngest if the following statements are true?**

- I. Aries is the oldest.
- II. Joseph is not the oldest.
- III. John is not the youngest.

- A. Aries
- B. John
- C. Joseph
- D. Neither of them are the youngest.



# TRUTH TELLERS & LIARS

- You are offered two cupcakes. One is poisoned and the other is safe to eat.
- You happen to be in a village full of knights (who always tell the truth) and knaves (who always lie), but you can't tell which is which by their appearance.
- You ask one of them, "Which cupcake is safe to eat?" To this he makes the following two statements.

"If I were a knave, I'd say the one on the right."  
"But I'd say the one on the left, if I were a knight."

## Which cupcake is safe to eat?

- **Assumption:** The person you ask knows which cupcake is which.



# TRUTH TELLERS & LIARS

- John tells the truth on Mondays, Thursdays, and Saturdays, but lies on all the other days of the week.
- One day he said, “I will tell the truth tomorrow.”

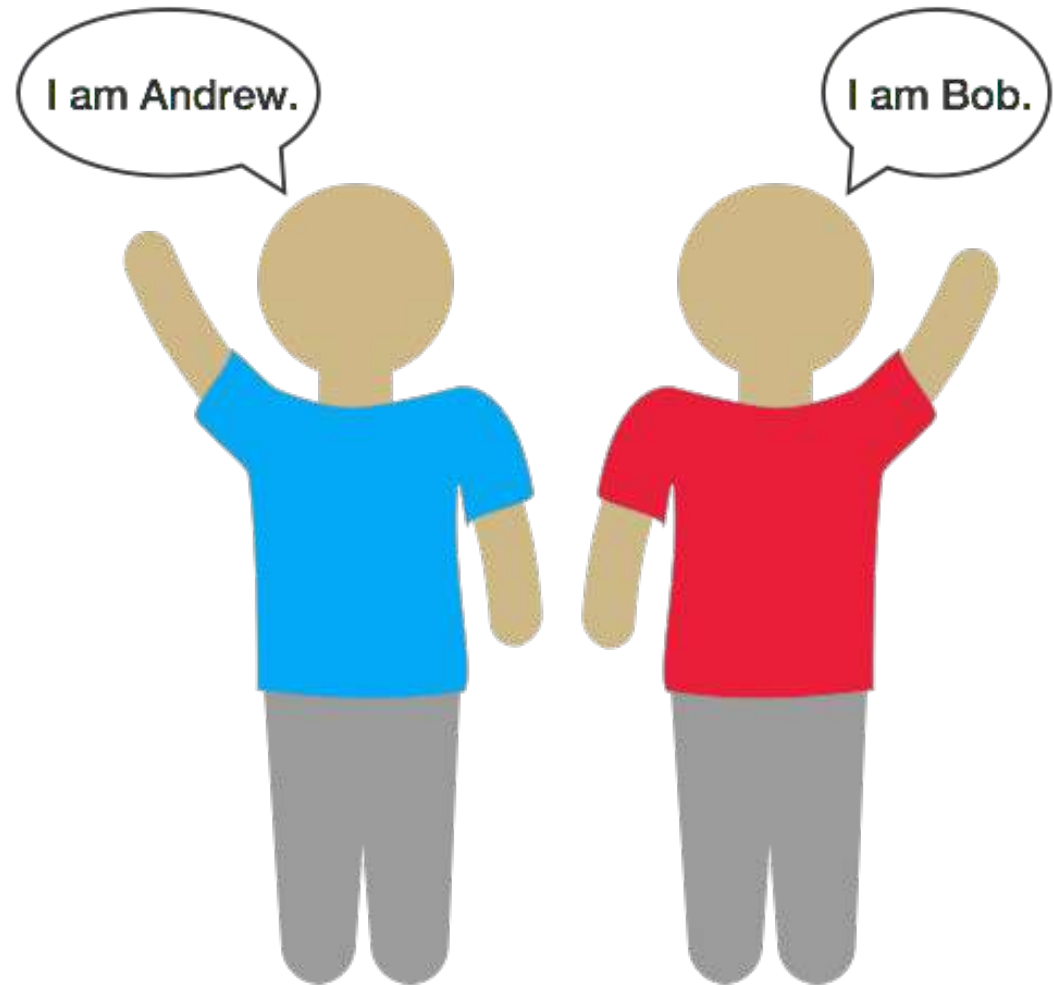
On which day of the week did he make this statement?



# TRUTH TELLERS & LIARS

- There are two men. One of them is wearing a red shirt, and the other is wearing a blue shirt. The two men are named Andrew and Bob, but we do not know which is Andrew and which is Bob.
- The guy in the blue shirt says, “I am Andrew.”  
The guy in the red shirt says, “I am Bob.”

If we know that at least one of them lied, then what color shirt is Andrew wearing?



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Thank you!